Electrical Technology

(EE-101F)

Contents

Introduction

Almost all electric power generation and most of the power transmission in the world is in the form of three-phase AC circuits. A three-phase AC system consists of three-phase generators, transmission lines, and loads.

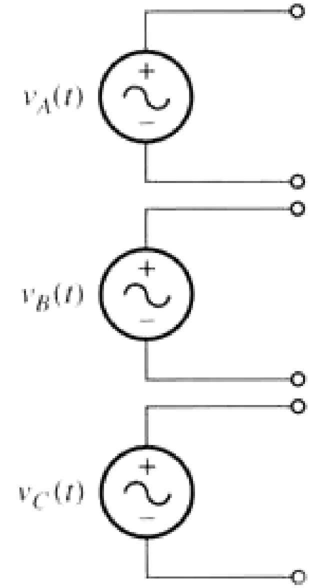
There are two major advantages of three-phase systems over a single-phase system:

1) More power per kilogram of metal form a three-phase machine;
2) Power delivered to a three-phase load is constant at all time, instead of pulsing as it does in a single-phase system.

The first three-phase electrical system was patented in 1882 by John Hopkinson -_British physicist, electrical engineer, Fellow of the Royal Society.

Three-phase voltages and currents

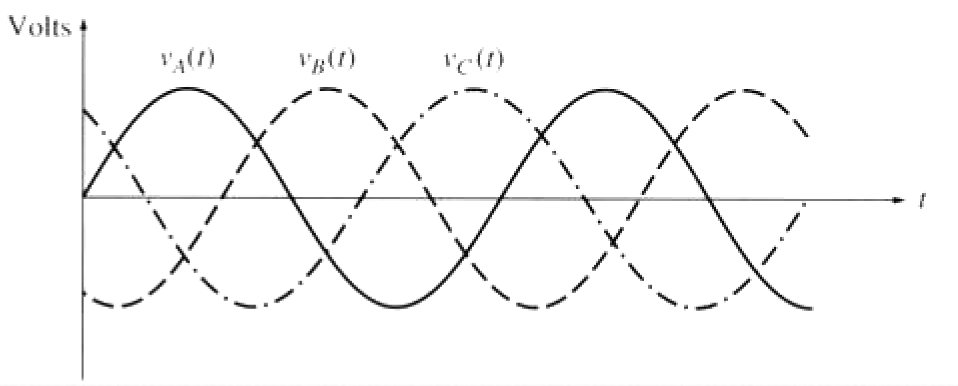
A three-phase generator consists of three single-phase generators with voltages of equal amplitudes and phase differences of 120^0.



$$v_A(t) = \sqrt{2}\ V \sin \omega t \text{ V}$$
$$\mathbf{V}_A = V \angle 0° \text{ V}$$

$$v_B(t) = \sqrt{2}\ V \sin (\omega t - 120°) \text{ V}$$
$$\mathbf{V}_B = V \angle -120° \text{ V}$$

$$v_C(t) = \sqrt{2}\ V \sin (\omega t - 240°) \text{ V}$$
$$\mathbf{V}_C = V \angle -240° \text{ V}$$



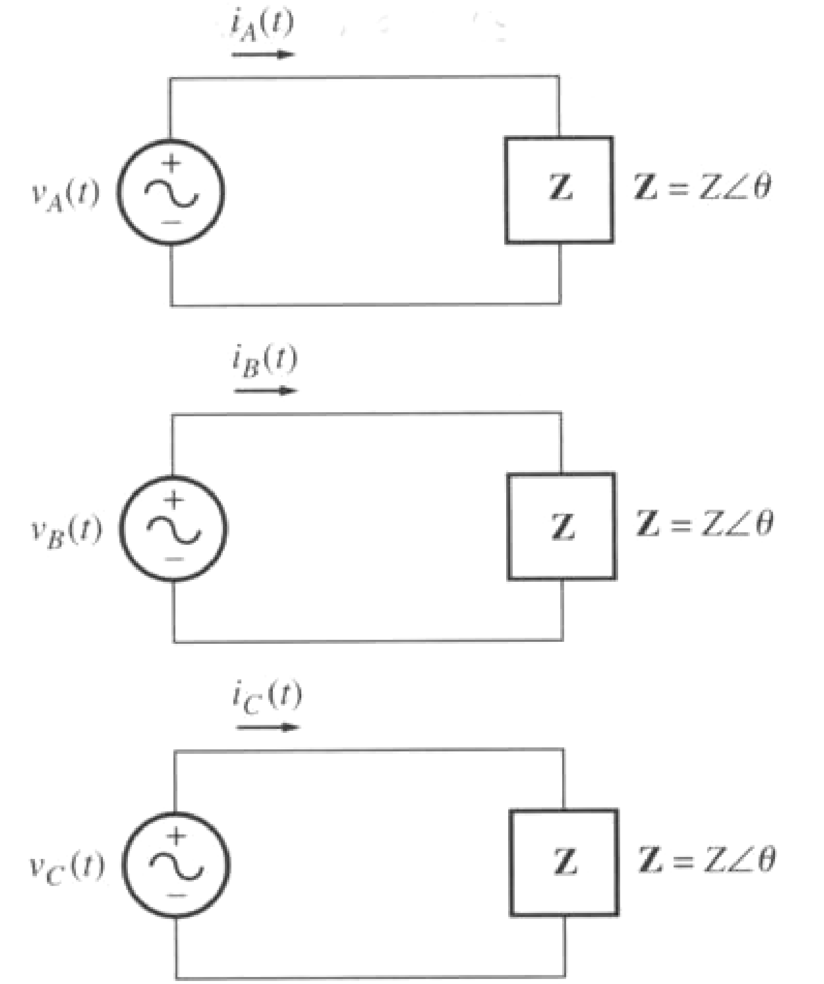
Three-phase voltages and currents

Each of three-phase generators can be connected to one of three identical loads.

This way the system would consist of three single-phase circuits differing in phase angle by 120^0.

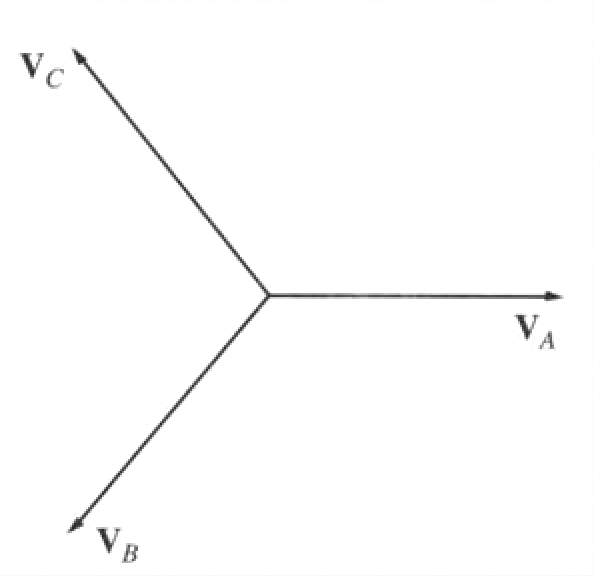
The current flowing to each load can be found as

$$I = \frac{V}{Z}$$



Three-phase voltages and currents

Therefore, the currents flowing in each phase are



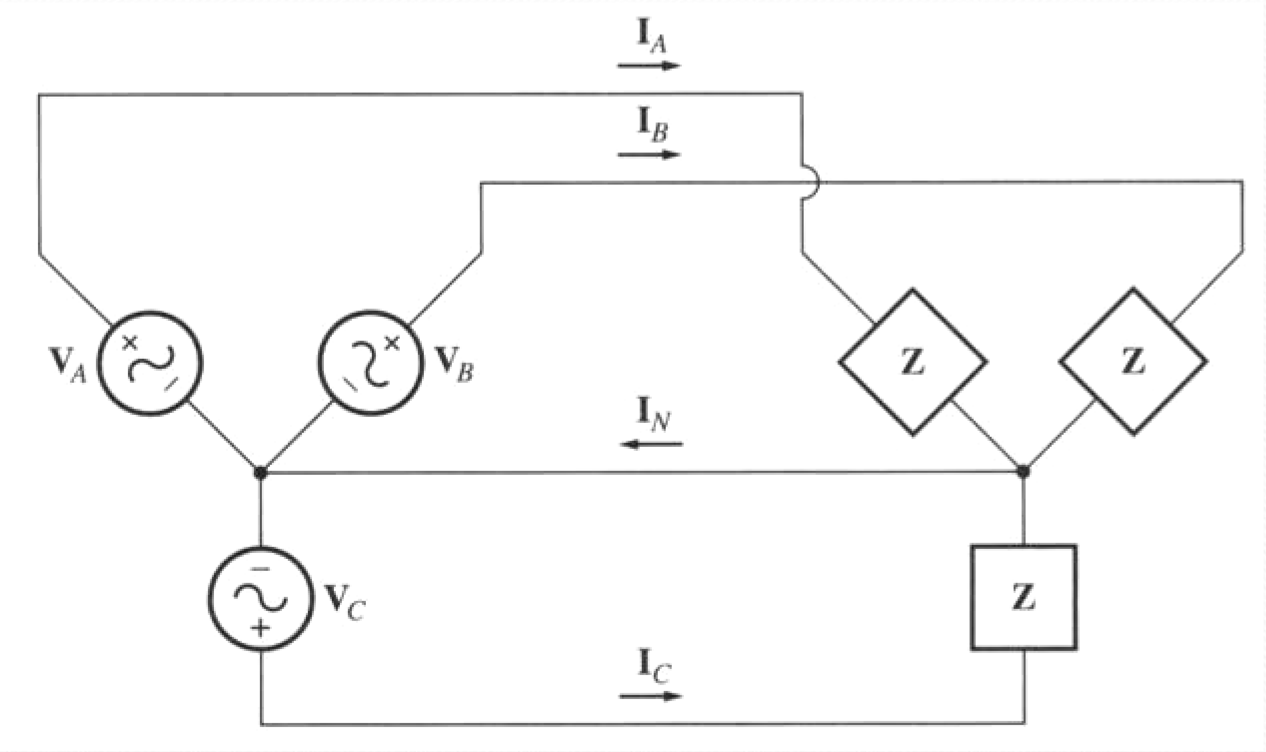
$$I_A = \frac{V\angle 0^0}{Z\angle\theta} = I\angle -\theta \qquad (.1)$$

$$I_B = \frac{V\angle -120^0}{Z\angle\theta} = I\angle -120-\theta \qquad .2)$$

$$I_A = \frac{V\angle -240^0}{Z\angle\theta} = I\angle -240-\theta \qquad .3)$$

Three-phase voltages and currents

We can connect the negative (ground) ends of the three single-phase generators and loads together, so they share the common return line (neutral).



Three-phase voltages and currents

The current flowing through a neutral can be found as

$$I_N = I_A + I_B + I_C = I\angle-\theta + I\angle-\theta-120^0 + I\angle-\theta-240^0$$

$$= I\cos(-\theta) + jI\sin(-\theta) + I\cos(-\theta-120^0) + jI\sin(-\theta-120^0) + I\cos(-\theta-240^0) + jI\sin(-\theta-240^0)$$

(.1)

$$= I\left[\cos(-\theta)+\cos(-\theta-120^0)+\cos(-\theta-240^0)\right] + jI\left[\sin(-\theta)+\sin(-\theta-120^0)+\sin(-\theta-240^0)\right]$$

$$= I\left[\cos(-\theta)+\cos(-\theta)\cos(120^0)+\sin(-\theta)\sin(120^0)+\cos(-\theta)\cos(240^0)+\sin(-\theta)\sin(240^0)\right]$$

$$+ jI\left[\sin(-\theta)+\sin(-\theta)\cos(120^0)-\cos(-\theta)\sin(120^0)+\sin(-\theta)\cos(240^0)-\cos(-\theta)\sin(240^0)\right]$$

Which is:

$$_N = I\left[\cos(-\theta)-\frac{1}{2}\cos(-\theta)+\frac{\sqrt{3}}{2}\sin(-\theta)-\frac{1}{2}\cos(-\theta)-\frac{\sqrt{3}}{2}\sin(-\theta)\right]$$

$$+ jI\left[\sin(-\theta)-\frac{1}{2}\sin(-\theta)+\frac{\sqrt{3}}{2}\cos(-\theta)-\frac{1}{2}\sin(-\theta)-\frac{\sqrt{3}}{2}\cos(-\theta)\right]$$

$$= 0$$

(.2)

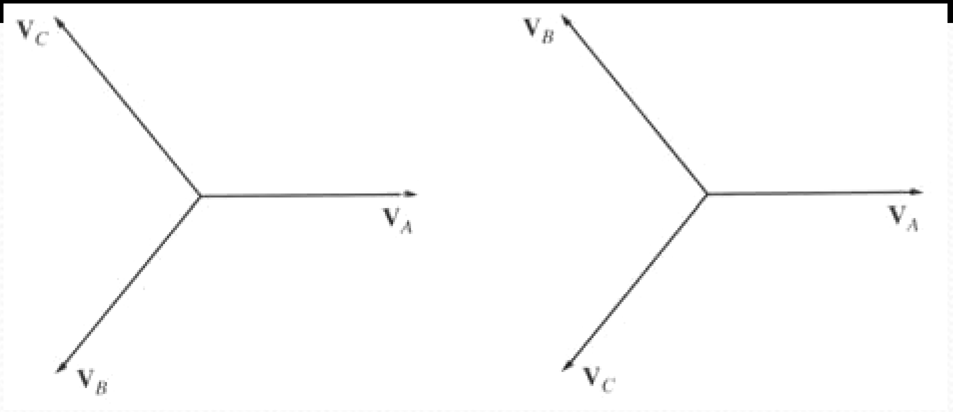
Three-phase voltages and currents

As long as the three loads are equal, the return current in the neutral is zero!

Such three-phase power systems (equal magnitude, phase differences of 120^0, identical loads) are called balanced.

In a balanced system, the neutral is unnecessary!

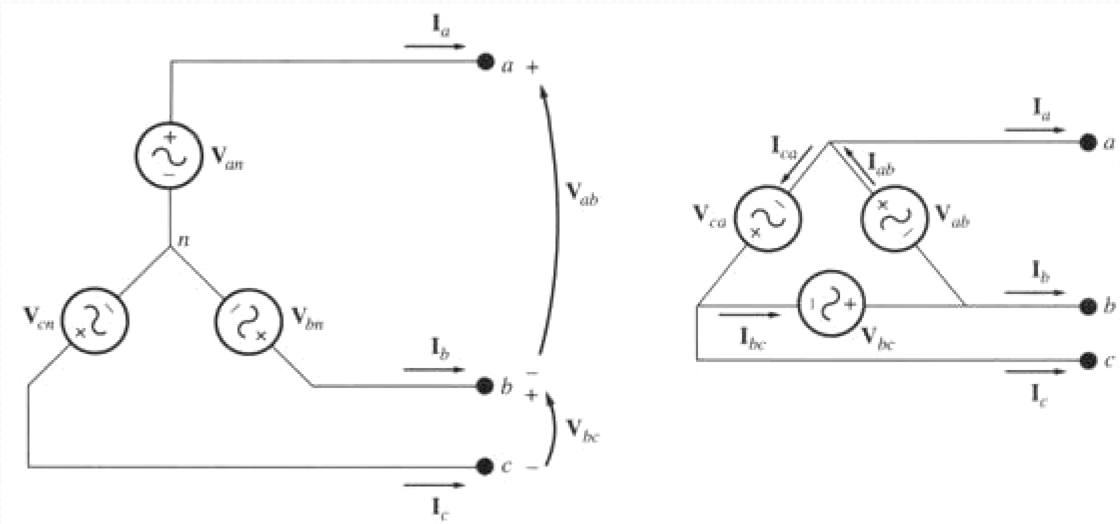
Phase Sequence is the order in which the voltages in the individual phases peak.



abc acb

Voltages and currents

There are two types of connections in three-phase circuits: Y and Δ.



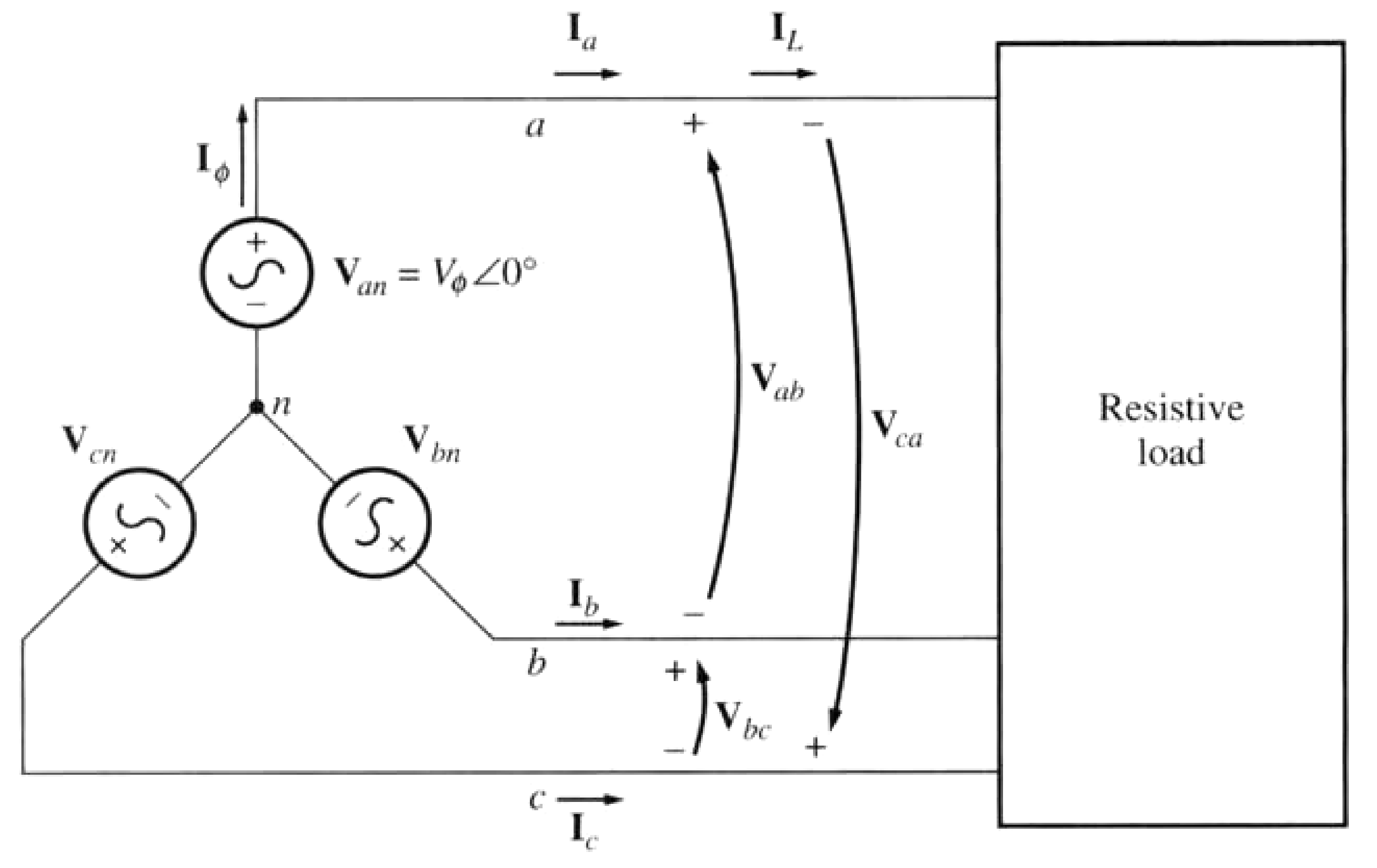
Each generator and each load can be either Y- or Δ-connected. Any number of Y- and Δ-connected elements may be mixed in a power system.

Phase quantities - voltages and currents in a given phase.
Line quantities – voltages between the lines and currents in the lines connected to the generators.

Voltages and currents

1. Y-connection

Assuming a resistive load…



Voltages and currents

1. Y-connection (cont)

$$V_{an} = V_{\phi}\angle 0^{0}$$
$$V_{bn} = V_{\phi}\angle -120^{0} \qquad (.1)$$
$$V_{cn} = V_{\phi}\angle -240^{0}$$

Since we assume a resistive load:

$$I_{a} = I_{\phi}\angle 0^{0}$$
$$I_{b} = I_{\phi}\angle -120^{0} \qquad (.2)$$
$$I_{c} = I_{\phi}\angle -240^{0}$$

Voltages and currents

1. Y-connection (cont.2)

The current in any line is the same as the current in the corresponding phase.

$$I_L = I_\phi \qquad (.1)$$

Voltages are:

$$V_{ab} = V_a - V_b = V_\phi \angle 0^0 - V_\phi \angle -120^0 = V_\phi - \left(-\frac{1}{2} V_\phi - j\frac{\sqrt{3}}{2} V_\phi \right) = \frac{3}{2} V_\phi + j\frac{\sqrt{3}}{2} V_\phi$$

$$= \sqrt{3} V_\phi \left(\frac{\sqrt{3}}{2} + j\frac{1}{2} \right) = \sqrt{3} V_\phi \angle 30^0 \qquad (.2)$$

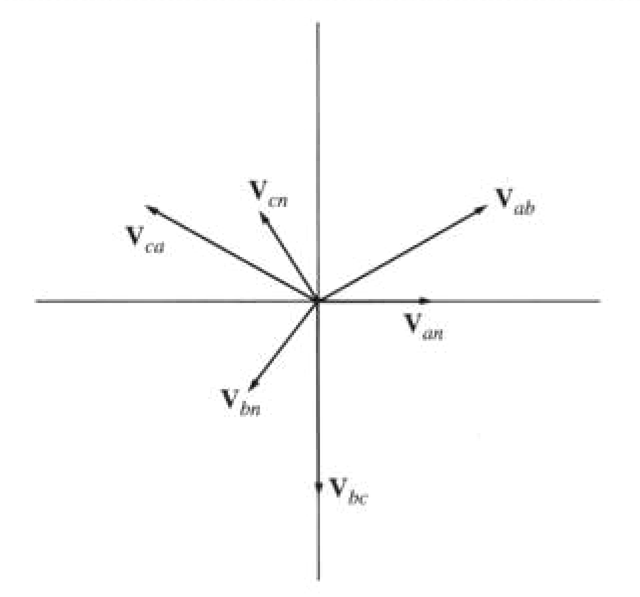
Voltages and currents

1. Y-connection (cont.3)

Magnitudes of the line-to-line voltages and the line-to-neutral voltages are related as:

$$V_{LL} = \sqrt{3} V_{\phi} \qquad (.1)$$

In addition, the line voltages are shifted by 30^0 with respect to the phase voltages.

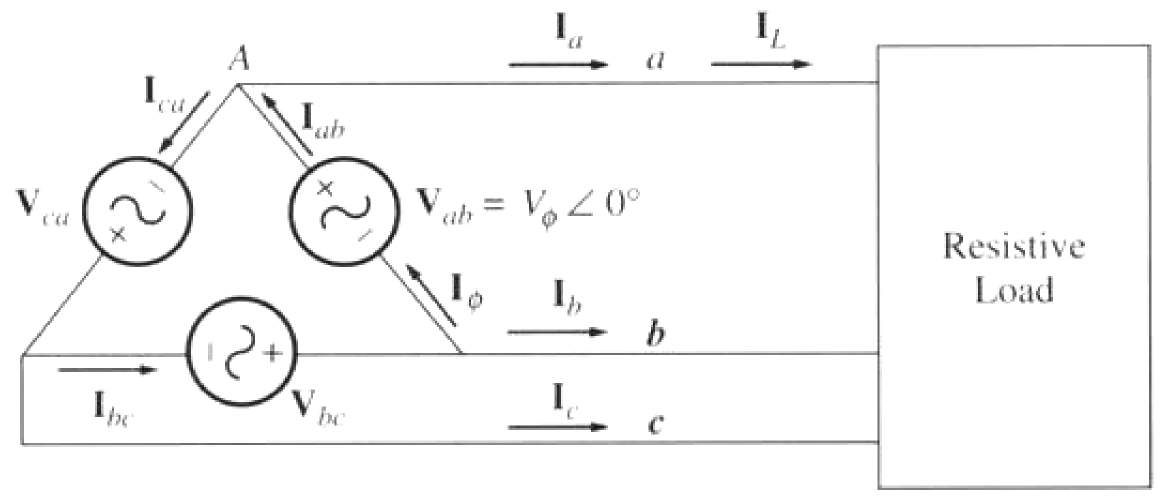


In a connection with *abc* sequence, the voltage of a line **leads** the phase voltage.

Voltages and currents

1. Δ connection

assuming a resistive load:



$$V_{ab} = V_\phi \angle 0^0$$
$$V_{bc} = V_\phi \angle -120^0 \quad (1)$$
$$V_{ca} = V_\phi \angle -240^0$$

$$I_{ab} = I_\phi \angle 0^0$$
$$I_{bc} = I_\phi \angle -120^0 \quad (.2)$$
$$I_{ca} = I_\phi \angle -240^0$$

Voltages and currents

1. Δ-connection (cont)

$$V_{LL} = V_{\phi} \qquad (.1)$$

The currents are:

$$I_a = I_{ab} - I_{ca} = I_{\phi}\angle 0^0 - I_{\phi}\angle 240^0 = I_{\phi} - \left(-\frac{1}{2} I_{\phi} + j\frac{\sqrt{3}}{2} I_{\phi} \right)$$

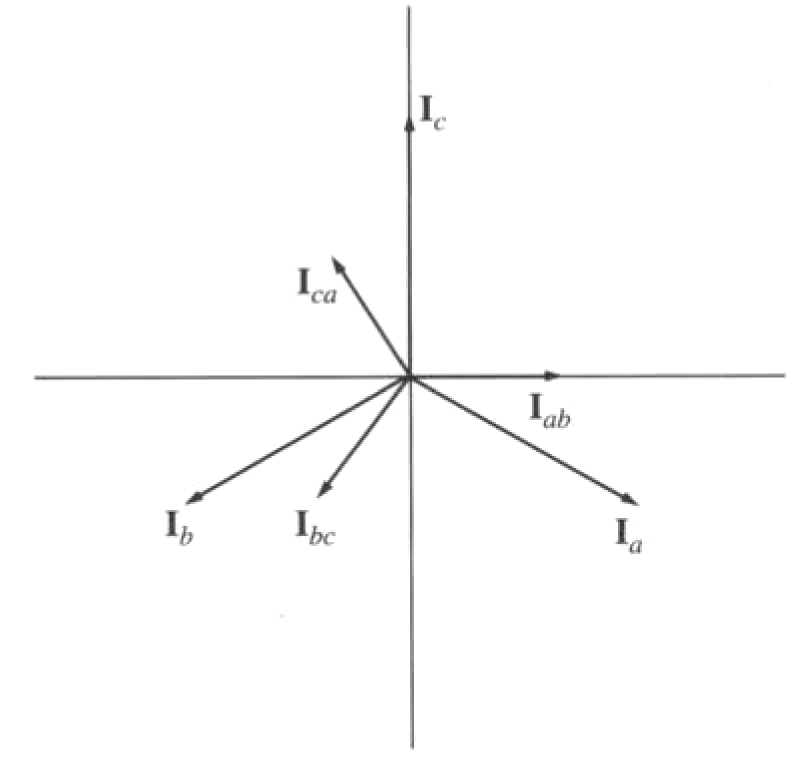
$$= \frac{3}{2} I_{\phi} - j\frac{\sqrt{3}}{2} I_{\phi} = \sqrt{3} I_{\phi} \left(\frac{\sqrt{3}}{2} - j\frac{1}{2} \right) = \sqrt{3} I_{\phi} \angle -30^0 \qquad (.2)$$

The magnitudes:

$$I_L = \sqrt{3} I_{\phi} \qquad (.3)$$

Voltages and currents

For the connections with the *abc* phase sequences, the current of a line **lags** the corresponding phase current by 30^0 (see Figure below).



For the connections with the *acb* phase sequences, the line current **leads** the corresponding phase current by 30^0.

Power relationships

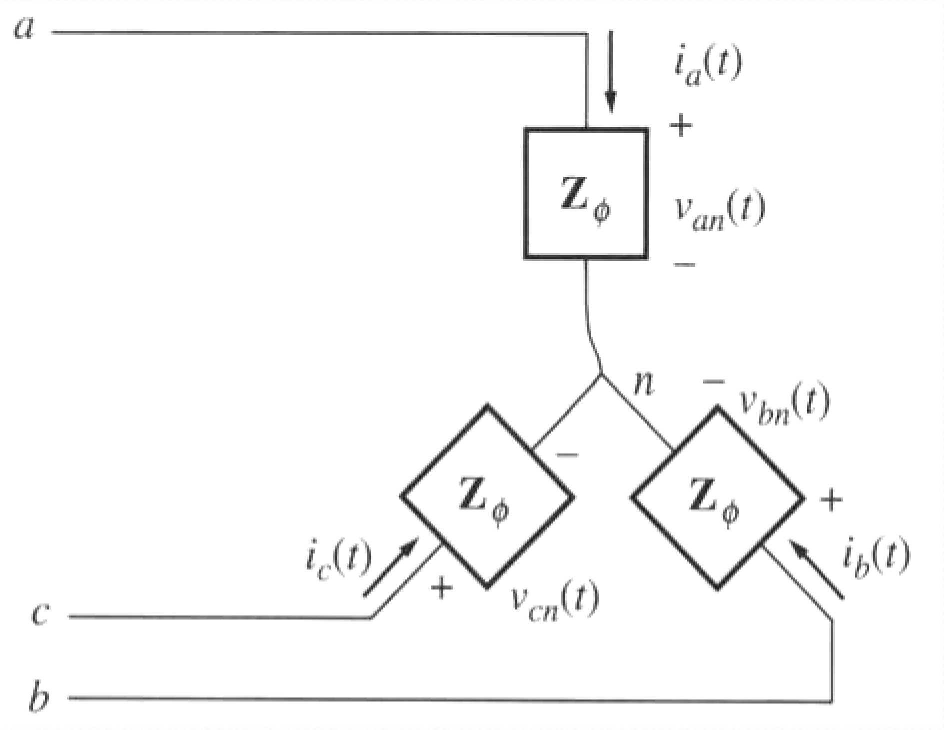
For a balanced Y-connected load with the impedance $Z_\phi = Z\angle\theta^0$:

and voltages:

$$v_{an}(t) = \sqrt{2}V\sin\omega t$$
$$v_{bn}(t) = \sqrt{2}V\sin(\omega t - 120^0)$$
$$v_{cn}(t) = \sqrt{2}V\sin(\omega t - 240^0) \quad (3.17.1)$$

The currents can be found:

$$i_a(t) = \sqrt{2}I\sin(\omega t - \theta)$$
$$i_b(t) = \sqrt{2}I\sin(\omega t - 120^0 - \theta)$$
$$i_c(t) = \sqrt{2}I\sin(\omega t - 240^0 - \theta) \quad (3.17.2)$$



Power relationships

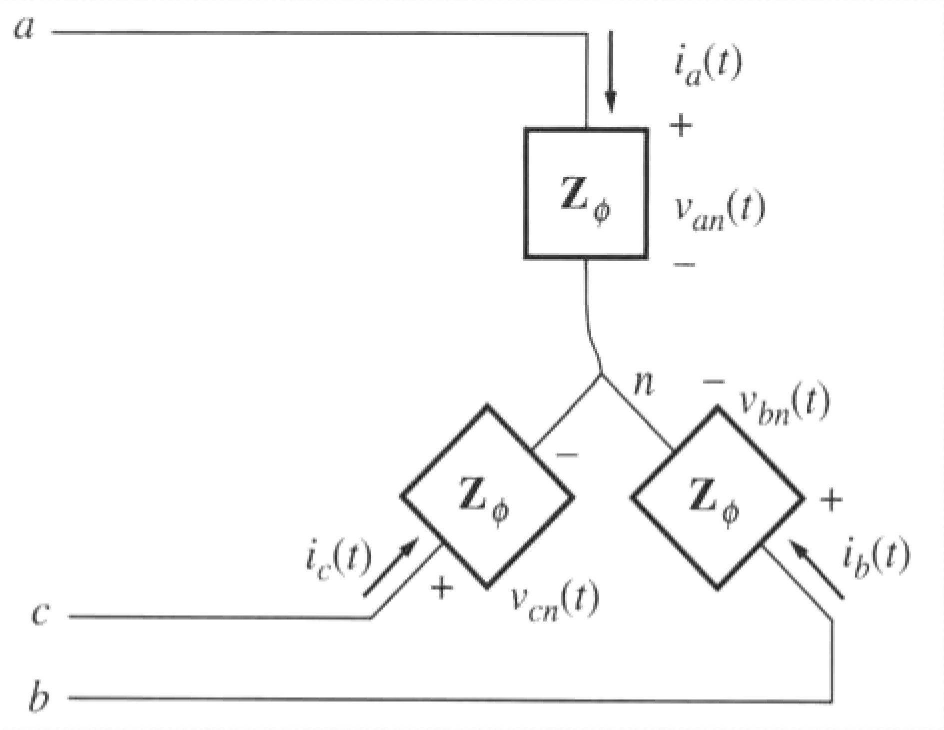
For a balanced Y-connected load with the impedance $Z_\phi = Z\angle\theta^0$:

and voltages:

$$v_{an}(t) = \sqrt{2}V \sin \omega t$$
$$v_{bn}(t) = \sqrt{2}V \sin(\omega t - 120^0)$$
$$v_{cn}(t) = \sqrt{2}V \sin(\omega t - 240^0) \quad (1)$$

The currents can be found:

$$i_a(t) = \sqrt{2}I \sin(\omega t - \theta)$$
$$i_b(t) = \sqrt{2}I \sin(\omega t - 120^0 - \theta)$$
$$i_c(t) = \sqrt{2}I \sin(\omega t - 240^0 - \theta) \quad (.2)$$



Power relationships

The instantaneous power is:

$$p(t) = v(t)i(t) \qquad (.1)$$

Therefore, the instantaneous power supplied to each phase is:

$$\begin{aligned} p_a(t) &= v_{an}(t)i_a(t) = 2VI\sin(\omega t)\sin(\omega t - \theta) \\ p_b(t) &= v_{bn}(t)i_b(t) = 2VI\sin(\omega t - 120^0)\sin(\omega t - 120^0 - \theta) \\ p_c(t) &= v_{cn}(t)i_c(t) = 2VI\sin(\omega t - 240^0)\sin(\omega t - 240^0 - \theta) \end{aligned} \qquad (.2)$$

Since

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right] \qquad (3)$$

Power relationships

Therefore

$$p_a(t) = VI\left[\cos\theta - \cos(2\omega t - \theta)\right]$$
$$p_b(t) = VI\left[\cos\theta - \cos(2\omega t - 240^0 - \theta)\right] \qquad (1)$$
$$p_c(t) = VI\left[\cos\theta - \cos(2\omega t - 480^0 - \theta)\right]$$

The total power on the load

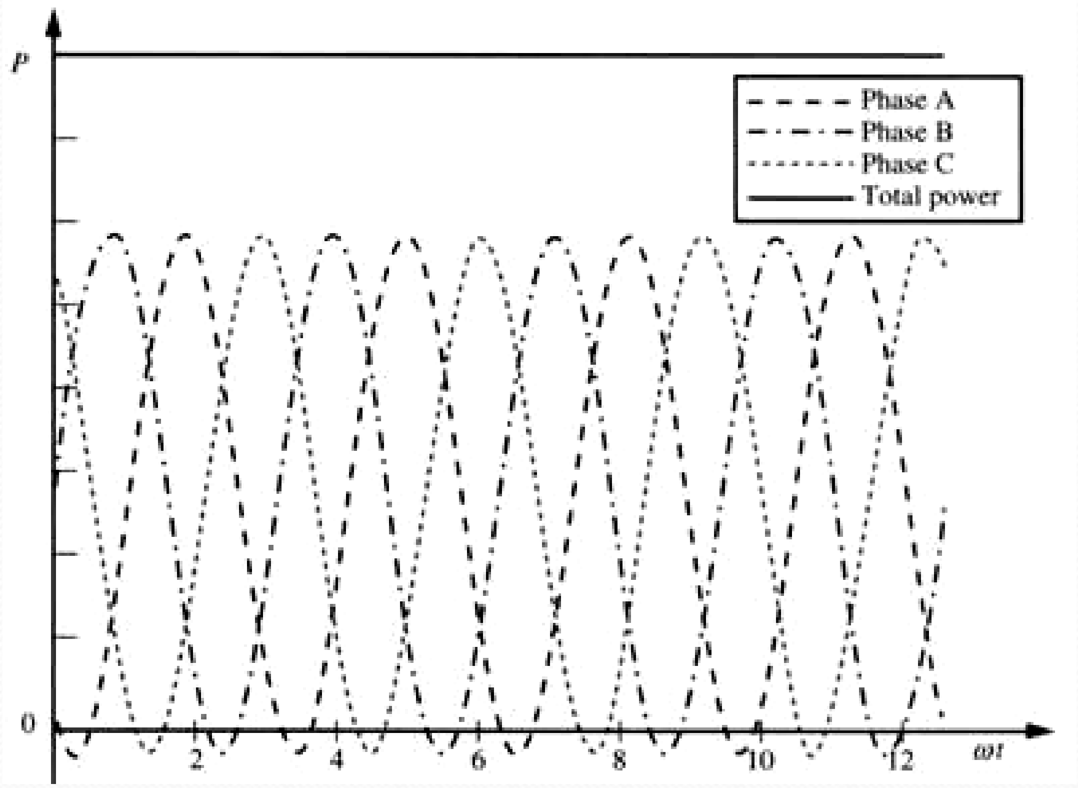
$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = 3VI\cos\theta \qquad (2)$$

The pulsing components cancel each other because of 120^0 phase shifts.

Power relationships

The instantaneous power in phases.

The total power supplied to the load is constant.



Power relationships

Phase quantities in each phase of a Y- or Δ-connection.

Real

$$P = 3V_\phi I_\phi \cos\theta = 3I_\phi{}^2 Z \cos\theta \quad (1)$$

Reactive

$$Q = 3V_\phi I_\phi \sin\theta = 3I_\phi{}^2 Z \sin\theta \quad (2)$$

Apparent

$$S = 3V_\phi I_\phi = 3I_\phi{}^2 Z \quad (3)$$

Note: these equations are valid for balanced loads only.

Power relationships

Line quantities: Y-connection:

Power consumed by a load: $P = 3V_{\phi} I_{\phi} \cos\theta$ (1)

Since for this load $I_L = I_{\phi} \;\; and \;\; V_{LL} = \sqrt{3} V_{\phi}$ (2)

Therefore: $P = 3 \frac{V_{LL}}{\sqrt{3}} I_L \cos\theta$ (3)

Finally: $P = \sqrt{3} V_{LL} I_L \cos\theta$ (4)

Note: these equations are valid for balanced loads only.

Power relationships

Line quantities: Δ-connection:

Power consumed by a load: $P = 3V_{\phi} I_{\phi} \cos\theta$ (1)

Since for this load $I_L = \sqrt{3} I_{\phi} \;\; and \;\; V_{LL} = V_{\phi}$ (.2)

Therefore: $P = 3\frac{I_L}{\sqrt{3}} V_{LL} \cos\theta$ (.3)

Finally: $P = \sqrt{3} V_{LL} I_L \cos\theta$ (4)

Same as for a Y-connected load!!

Note: these equations were derived for a balanced load.

Power relationships

Line quantities: Y- and Δ-connection

Reactive power

$$Q = \sqrt{3} V_{LL} I_L \sin\theta \quad (1)$$

Apparent power

$$S = \sqrt{3} V_{LL} I_L \quad (2)$$

Note: θ is the angle between the **phase voltage** and the **phase current – the impedance angle.**

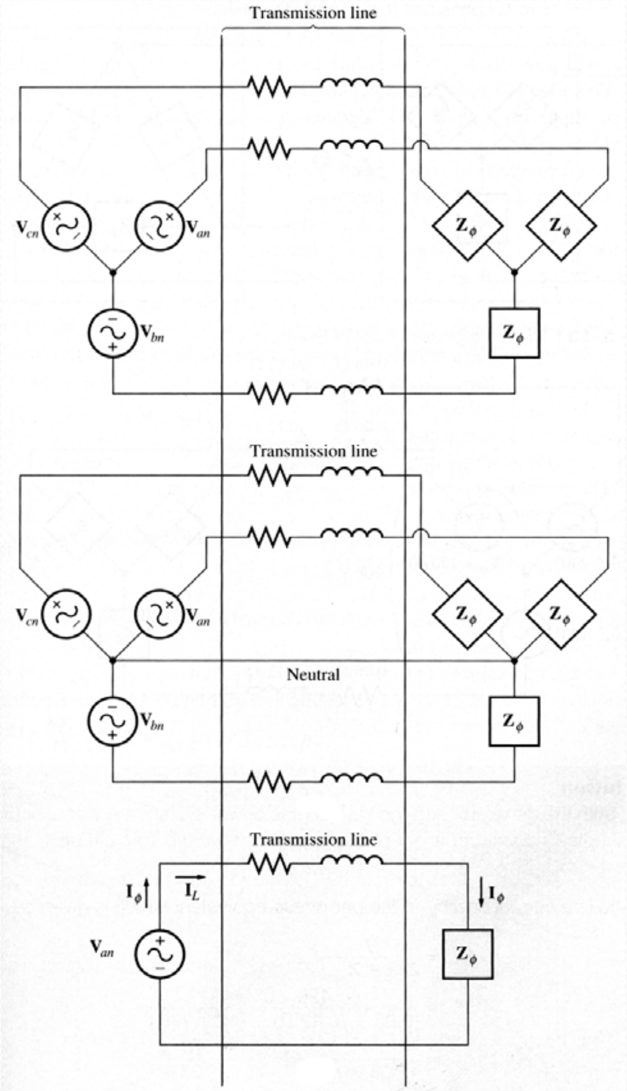
Analysis of balanced systems

We can determine voltages, currents, and powers at various points in a balanced circuit.

Consider a Y-connected generator and load via three-phase transmission line.

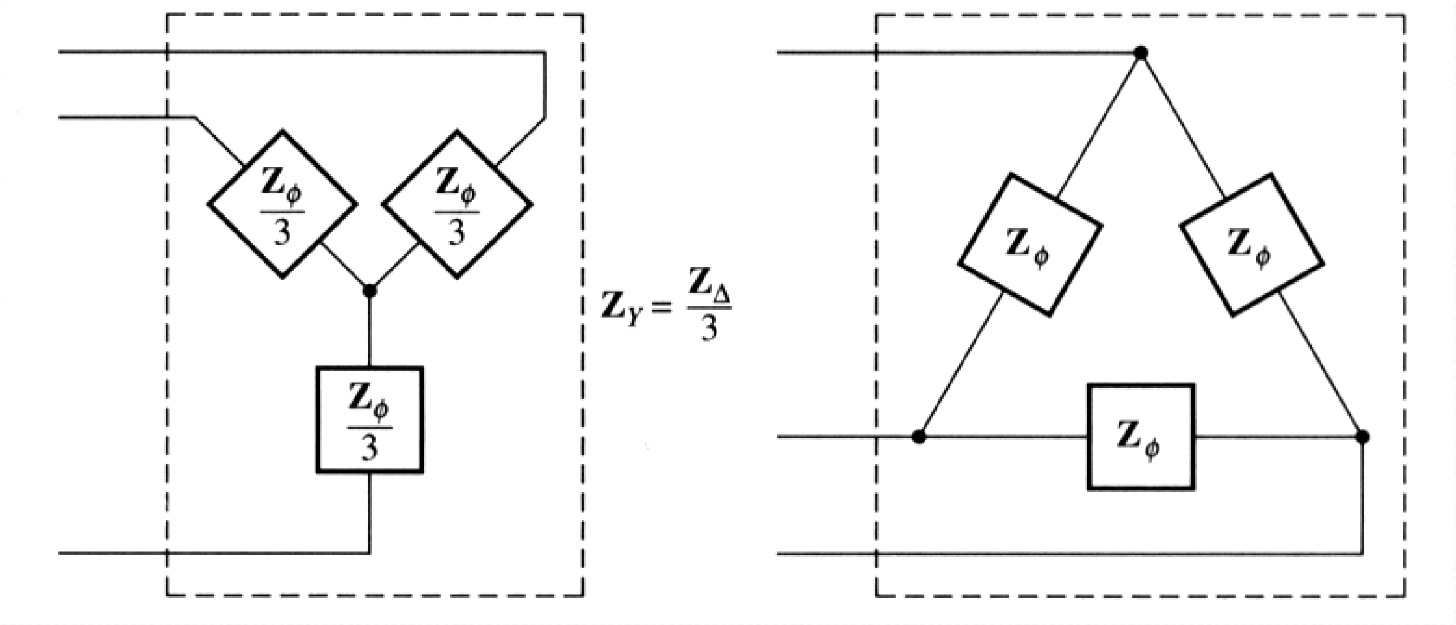
For a balanced Y-connected system, insertion of a neutral does not change the system.
All three phases are identical except of 120^0 shift. Therefore, we can analyze a single phase (per-phase circuit).

Limitation: not valid for Δ-connections…



Analysis of balanced systems

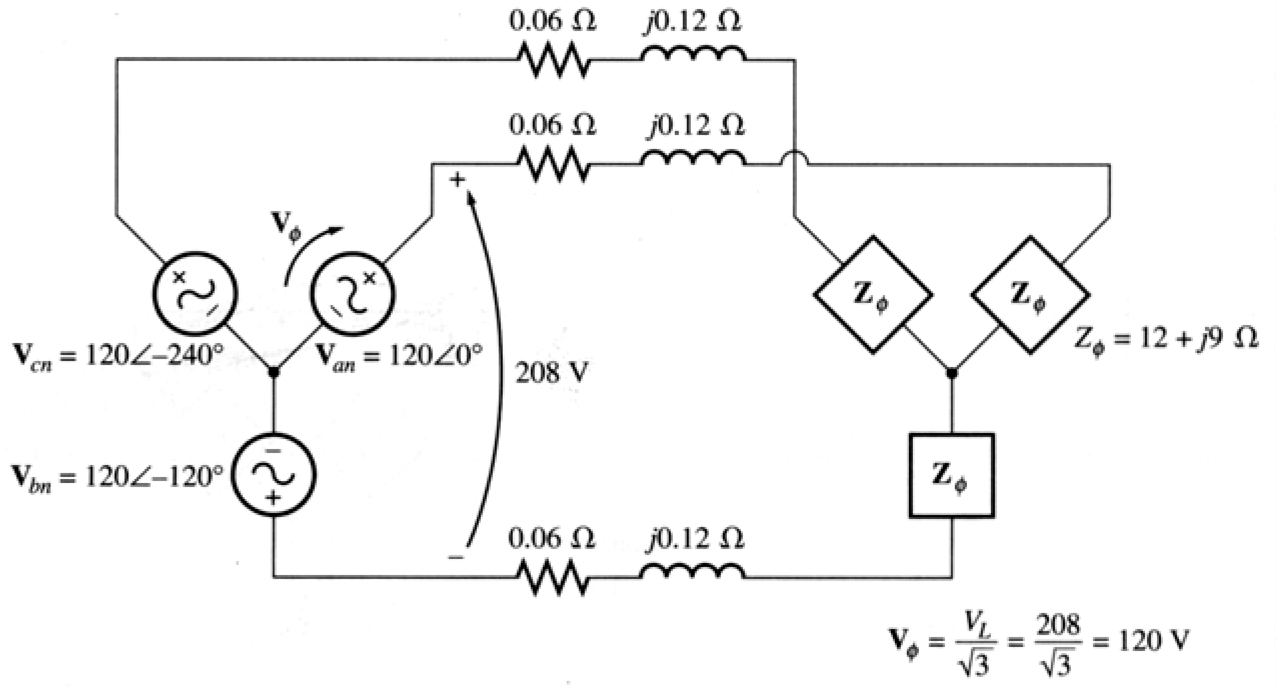
A Δ-connected circuit can be analyzed via the transform of impedances by the Y-Δ transform. For a balanced load, it states that a Δ-connected load consisting of three equal impedances *Z* is equivalent to a Y-connected load with the impedances *Z*/3. This equivalence implies that the voltages, currents, and powers supplied to both loads would be the same.



Analysis of balanced systems: Ex

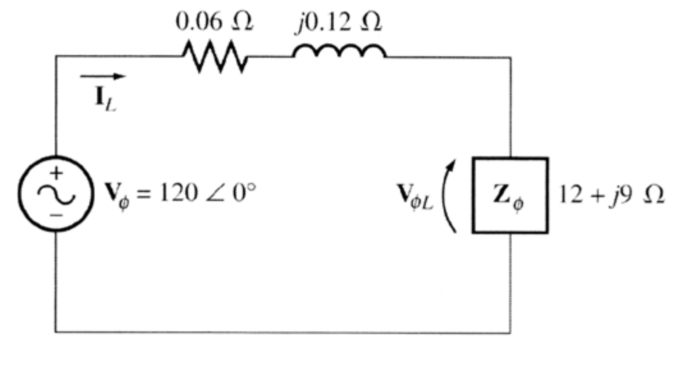
Example 3-1:
for a 208-V three-phase ideally balanced system, find:

a) the magnitude of the line current I_L;
b) The magnitude of the load's line and phase voltages V_{LL} and $V_{\phi L}$;
c) The real, reactive, and the apparent powers consumed by the load;
d) The power factor of the load.



Analysis of balanced systems: Ex

Both, the generator and the load are Y-connected, therefore, it's easy to construct a per-phase equivalent circuit…



Btw: is the load inductive or capacitive???

a) The line current:

$$I_L = \frac{V}{Z_L + Z_{load}} = \frac{120\angle 0^0}{(0.06 + j0.12) + (12 + j9)} = \frac{120\angle 0^0}{12.06 + j9.12} = \frac{120\angle 0^0}{15.12\angle 37.1^0} = 7.94\angle -37.1^0 \ \ A$$

b) The phase voltage on the load:

$$V_{\phi L} = I_{\phi L} Z_{\phi L} = (7.94\angle -37.1^0)(12 + j9) = (7.94\angle -37.1^0)(15\angle 36.9^0) = 119.1\angle -0.2^0 \ \ V$$

The magnitude of the line voltage on the load:

$$V_{LL} = \sqrt{3} V_{\phi L} = 206.3 \ \ V$$

Analysis of balanced systems: Ex

c) The real power consumed by the load:

$$P_{load} = 3V_{\phi}I_{\phi}\cos\theta = 3\cdot 119.1\cdot 7.94\cos 36.9^{0} = 2270\ W$$

The reactive power consumed by the load:

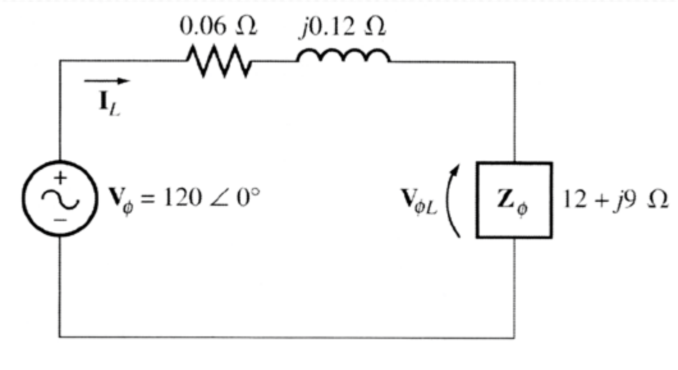
$$Q_{load} = 3V_{\phi}I_{\phi}\sin\theta = 3\cdot 119.1\cdot 7.94\sin 36.9^{0} = 1702\ \text{var}$$

The apparent power consumed by the load:

$$S_{load} = 3V_{\phi}I_{\phi} = 3\cdot 119.1\cdot 7.94 = 2839\ VA$$

d) The load power factor:

$$PF_{load} = \cos\theta = \cos 36.9^{0} = 0.8\ -lagging$$

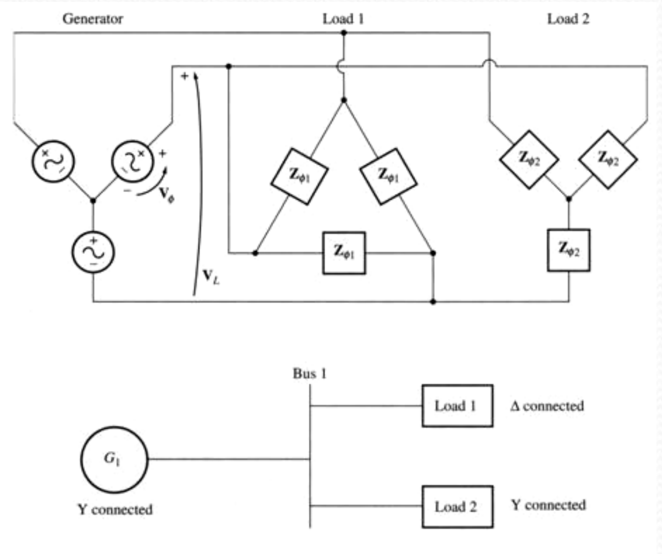


Using the power triangle

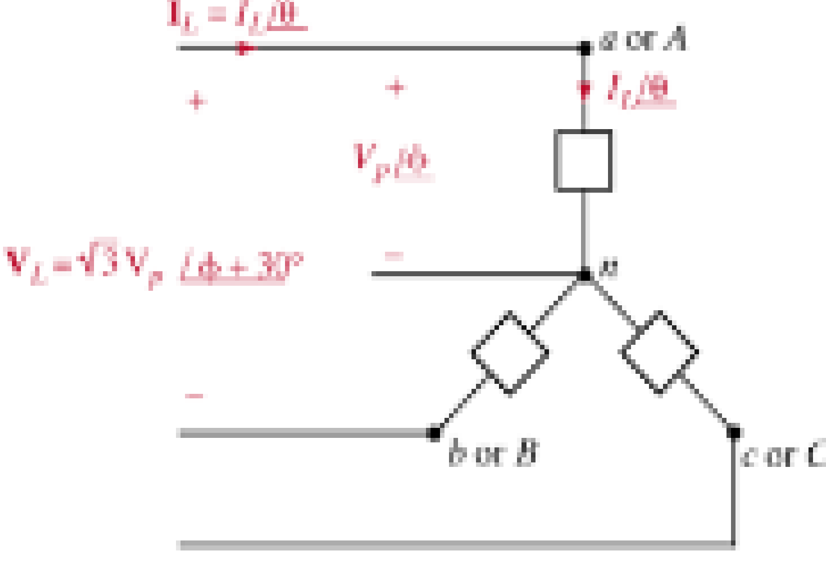
If we can neglect the impedance of the transmission line, an important simplification in the power calculation is possible…

If the generator voltage in the system is known, then we can find the current and power factor at any point in the system as follows:

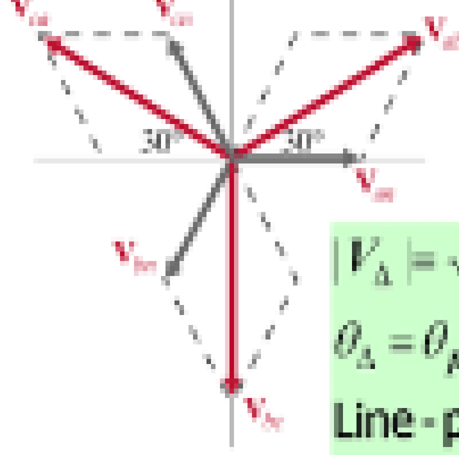
1. The line voltages at the generator and the loads will be identical since the line is lossless.
2. Real and reactive powers on each load.
3. The total real and reactive powers supplied to all loads from the point examined.
4. The system power factor at that point using the power triangle relationship.
5. Line and phase currents at that point.



We can treat the line voltage as constant and use the power triangle method to quickly calculate the effect of adding a load on the overall system and power factor.



(a)



(b)

$|V_\Delta| = \sqrt{3}\,|V_{phase}|$

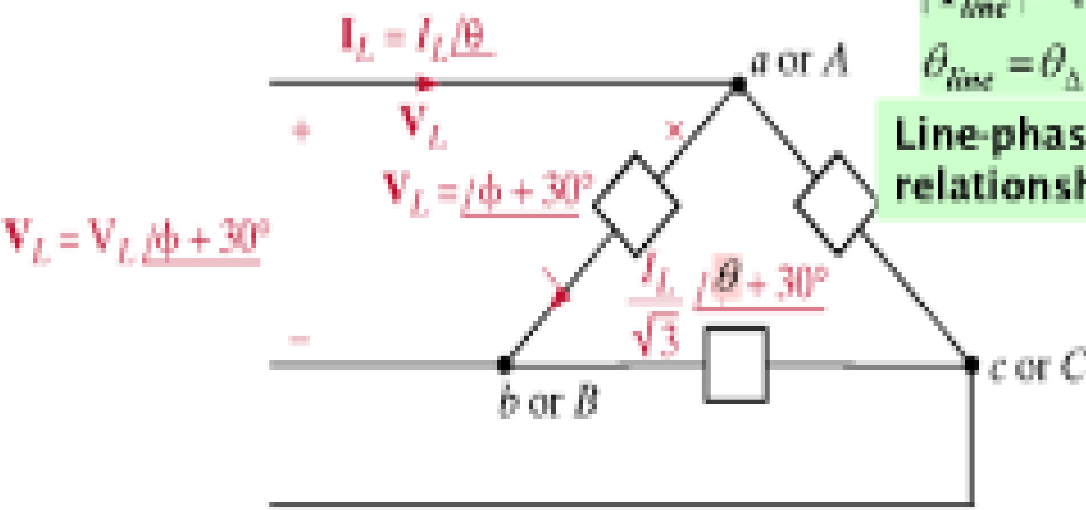
$\theta_\Delta = \theta_{phase} + 30°$

Line - phase voltage relationship

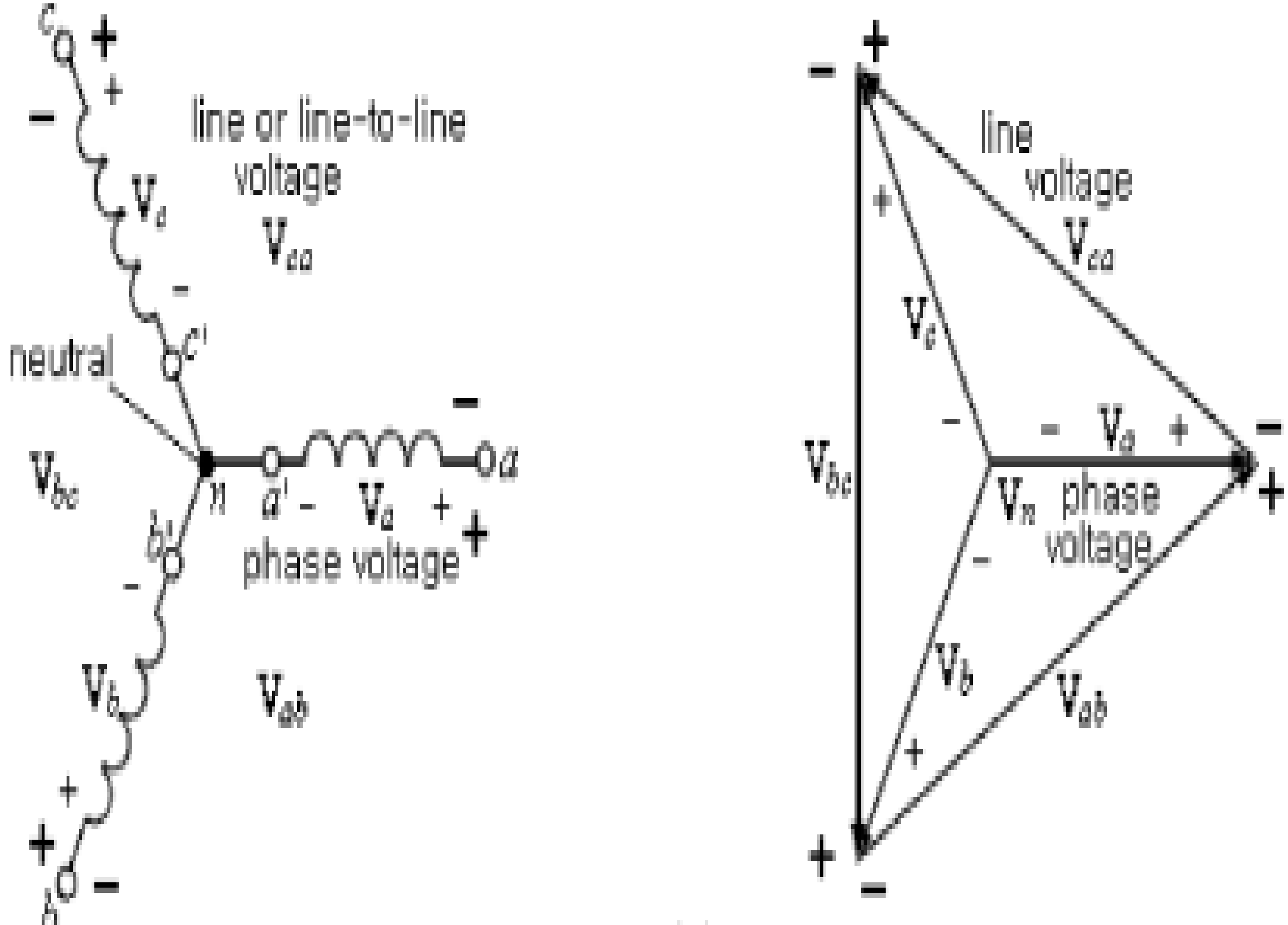
$|I_{line}| = \sqrt{3}\,|I_\Delta|$

$\theta_{line} = \theta_\Delta - 30°$

Line-phase current relationship



(b)



(a) A Y-connected 3-ϕ source

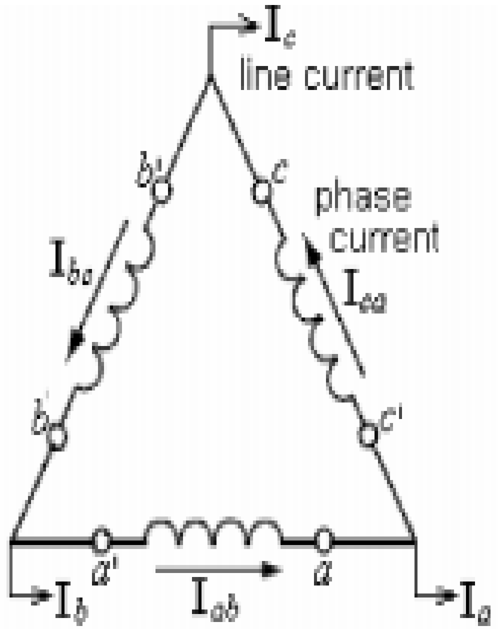
(b) The phase and line voltage phasor diagram for a Y-connected 3-ϕ source

A Y-connected 3-ϕ source and its voltage phasor diagram

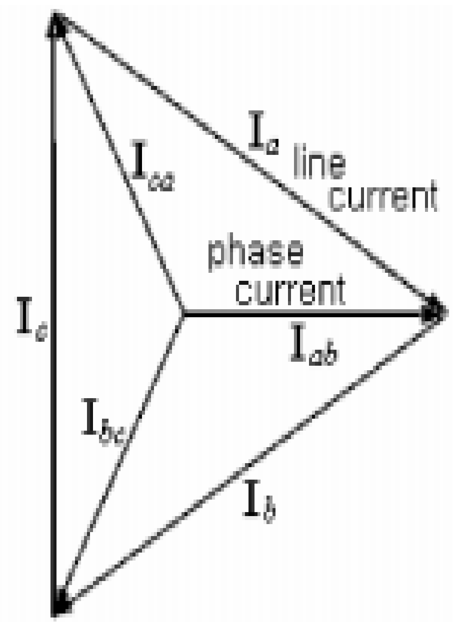
$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b = \sqrt{3}\, V_Y \angle 30^\circ$$
$$\mathbf{V}_{bc} = \mathbf{V}_b - \mathbf{V}_c = \sqrt{3}\, V_Y \angle -90^\circ$$
$$\mathbf{V}_{ca} = \mathbf{V}_c - \mathbf{V}_a = \sqrt{3}\, V_Y \angle +150^\circ$$

the relationship between the *phase* (or *line-to-neutral*) *voltages* and the *line* (or *line-to-line*) *voltages* can be written as follows:

$$V_l = \sqrt{3}\, V_Y \qquad \text{and} \qquad \mathbf{I}_l = \mathbf{I}_Y$$



(a) A Δ-connected 3-ϕ source



(b) The phase and line current phasor diagram for a Δ-connected 3-ϕ source

A Δ-connected 3-ϕ source and its current phasor diagram

the relationship between the *phase currents* and the *line currents* can be written as follows:

$$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca} = \sqrt{3}\, I_\Delta \angle -30^\circ$$

$$\mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab} = \sqrt{3}\, I_\Delta \angle -150^\circ$$

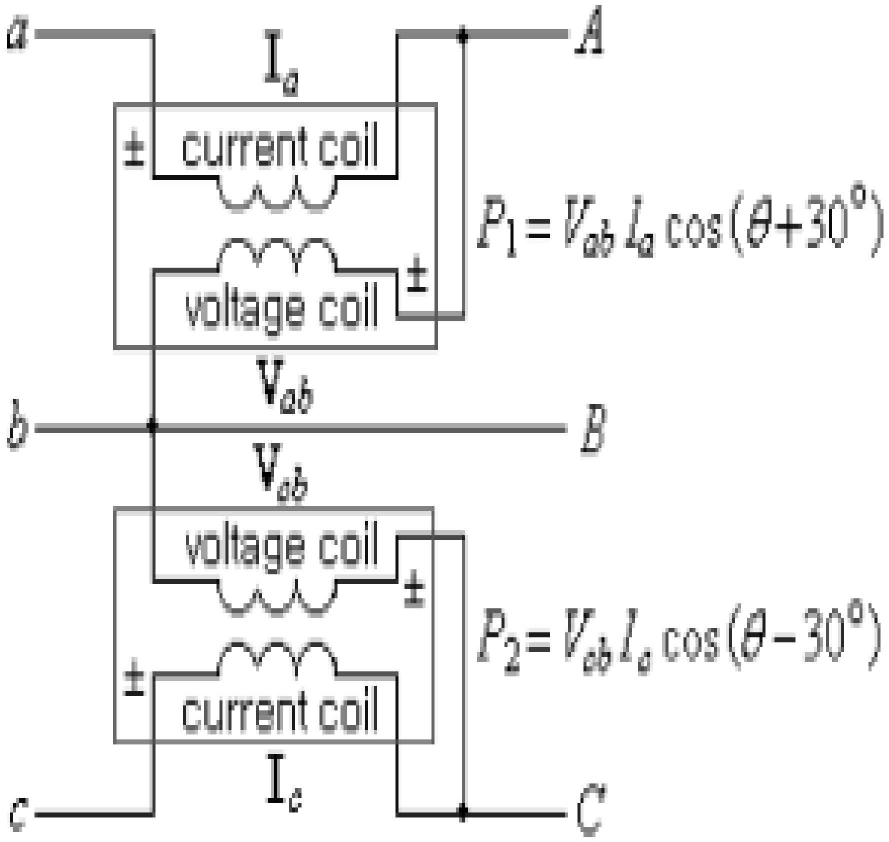
$$\mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc} = \sqrt{3}\, I_\Delta \angle +90^\circ$$

Note that for a Δ-connected three-phase source, the amplitudes of line currents are $\sqrt{3}$ times that of phase currents and the line voltage is the same as the phase voltage:

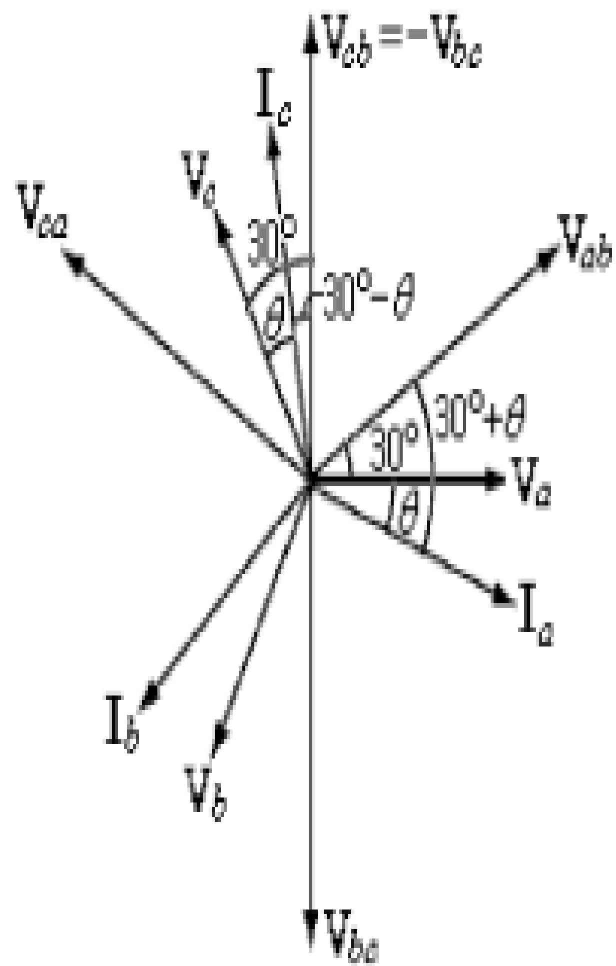
$$I_l = \sqrt{3}\, I_\Delta \qquad \text{and} \qquad \mathbf{V}_l = \mathbf{V}_\Delta$$

Measurement of Three-Phase Power

three-phase power can be measured using just two wattmeters.



(a) Two-wattmeter connection to measure a three-phase AC power



(b) The phasor diagrams for voltages/currents

- From the connections shown in fig (a) above , the readings of the two wattmeters can be written as

$$P_1 = V_{ab} I_a \cos(\theta + 30^\circ)$$
$$P_2 = V_{cb} I_c \cos(\theta - 30^\circ)$$

From the phasor diagram shown in fig (b) above, the sum of these two readings gives the three phase active power

$$P_1 + P_2 = V_{ab} I_a \cos(\theta + 30^\circ) + V_{cb} I_c \cos(\theta - 30^\circ)$$
$$= V_l I_l \cos(\theta + 30^\circ) + V_l I_l \cos(\theta - 30^\circ)$$
$$= 2 V_l I_l \cos\theta \cos 30^\circ = \sqrt{3}\, V_l I_l \cos\theta \equiv P_{\text{Total}}$$